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Probabilistic dipole inversion for adaptive quantitative susceptibility mapping

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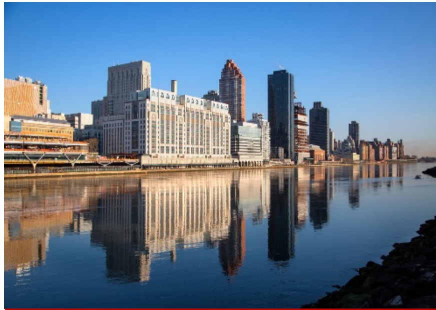


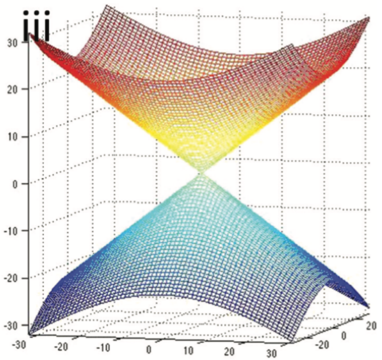
Weill Cornell Medicine

Quantitative susceptibility mapping (QSM)

χ: tissue susceptibility
b: magnetic field
d: dipole kernel
n: measurement noise

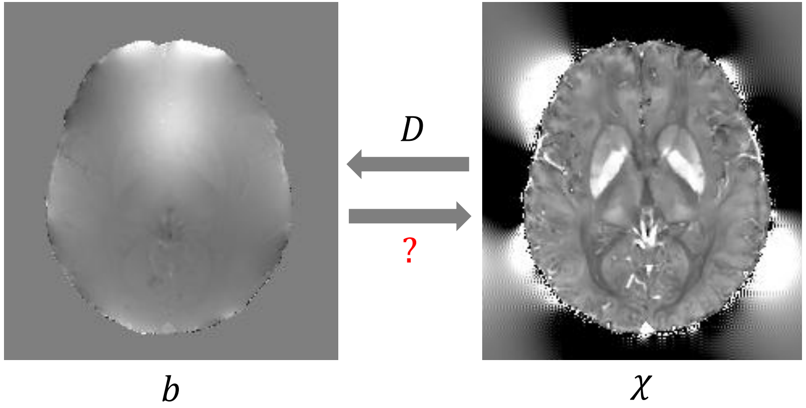
(Image space) (K-space)

$$b = \chi * d + n \quad \xleftrightarrow{D = \mathcal{F}[d]} \quad b = F^H D F \chi + n$$



The zero cone of D in k-space

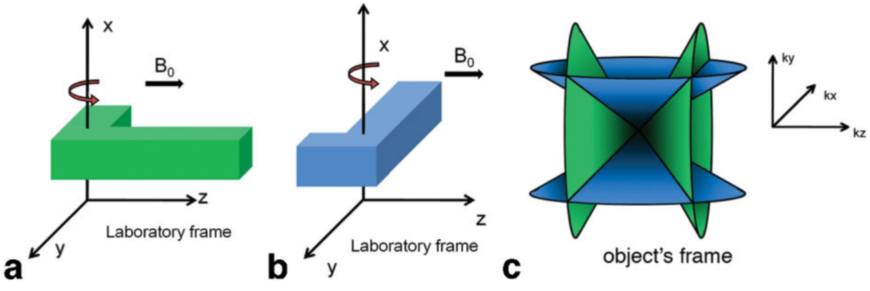
Wang, Yi, and Tian Liu. *Magnetic resonance in medicine* 73.1 (2015): 82-101.



COSMOS and MEDI

COSMOS

Multi-orientation scans, golden standard QSM



Rotation1

Rotation 2

Zeroes located on a pair of cone surfaces

Liu, Tian, et al. *Magnetic Resonance in Medicine* 61.1 (2009): 196-204.

MEDI

Single-orientation scan, clinically feasible QSM

$$p(b|\chi) = \mathcal{N}\left(b \middle| F^H D F\chi, \Sigma_{b|\chi}\right), p(\chi) \propto e^{-\lambda\|M\nabla\chi\|_1}$$

$$\chi_{\mathrm{MAP}} = \arg\min_{\chi} \log p(b|\chi) + \log p(\chi)$$

Binary-valued weighting matrix M (three spatial directions)

Liu, Jing, et al. *Neuroimage* 59.3 (2012): 2560-2568.

Motivation: fitting susceptibility distributions

- Given $p(\chi)$ and $p(b|\chi)$, solving $p(\chi|b)$?
- Traditional approximate inference methods: MCMC, VI. Need to run on each subject.
- Can we learn a general distribution $p_{\text{data}}(\chi|b)$ for any given b?
- Introduce parametrized distributions $q_\psi(\chi|b)$, learn ψ so that $q_\psi(\chi|b) \approx p_{\text{data}}(\chi|b)$ (amortized optimization).

COSMOS dataset and modeling

$\left(\chi^{(1)}, b^{(1)}\right), \dots, \left(\chi^{(N)}, b^{(N)}\right)$ sampled from $p_{\mathrm{data}}(\chi|b)$

↓ empirical distribution

$$\hat{p}_{\mathrm{data}}(\chi|b) = \frac{1}{N}\Sigma_{i=1}^{\mathrm{N}}\mathbf{1}[\chi = \chi^{(i)}|b = b^{(i)}]$$

↓ $KL[\hat{p}_{\mathrm{data}}(\chi|b) \parallel q_\psi(\chi|b)]$

$$\frac{1}{N}\Sigma_{i=1}^{\mathrm{N}} - \log q_\psi\left(\chi^{(i)}\middle|b^{(i)}\right) + \sout{H(\hat{p}_{\mathrm{data}})}$$

MEDI dataset and modeling

Only $b^{(1)}, \ldots, b^{(M)}$ are given. $p(b|\chi)$ and $p(\chi)$.

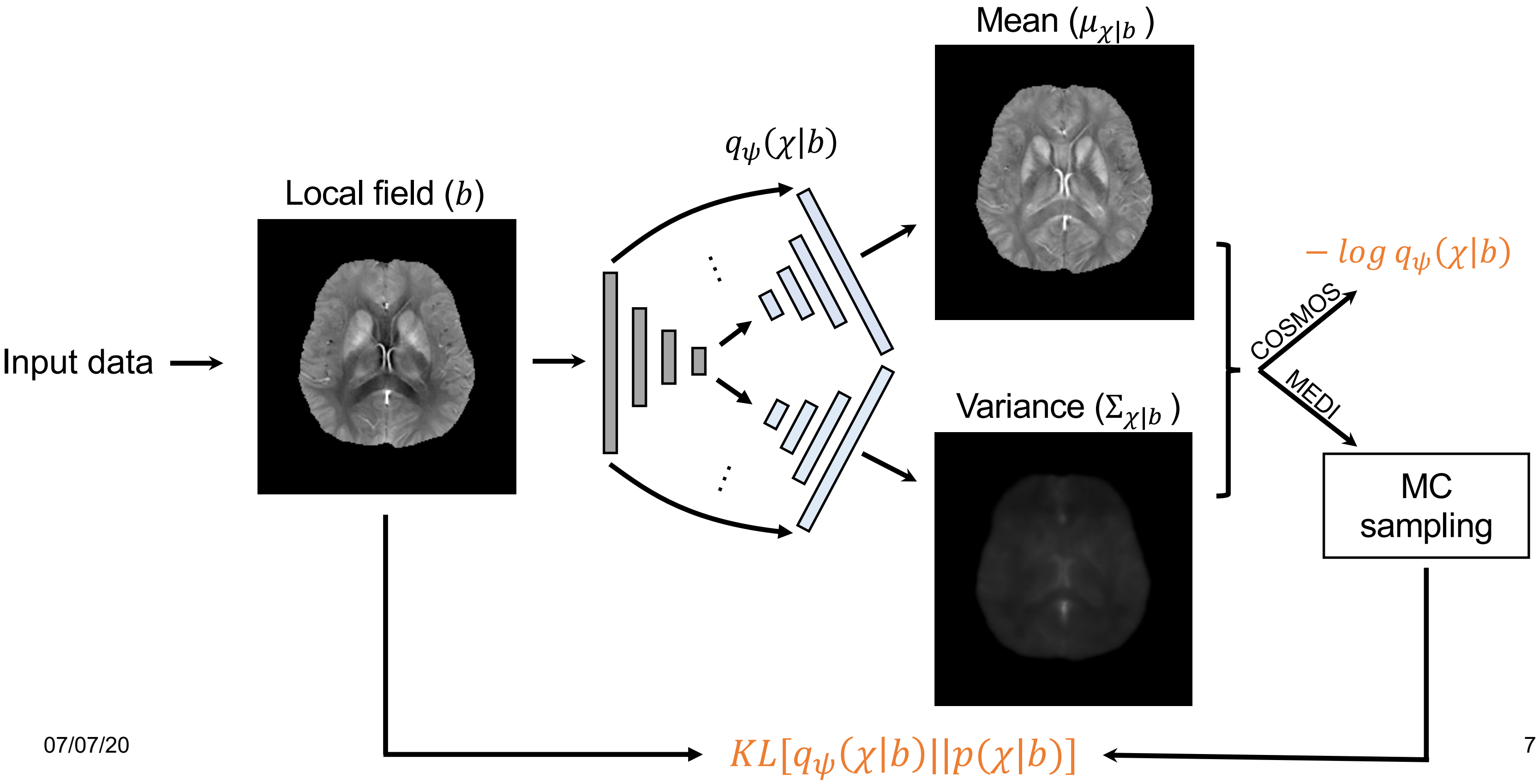
↓ $KL[q_\psi(\chi|b) \parallel p(\chi|b)]$

$$KL[q_\psi(\chi|b) \parallel p(\chi)] - \mathbb{E}_{q_\psi}[\log p(b|\chi)]$$

↓ Amortized formulation

$$\underbrace{\Sigma_{i=1}^{M} KL[q_\psi(\chi|b^{(i)}) \parallel p(\chi)]}_{\text{Regularization}} - \underbrace{\mathbb{E}_{q_\psi}[\log p(b^{(i)}|\chi)]}_{\text{Likelihood}}$$

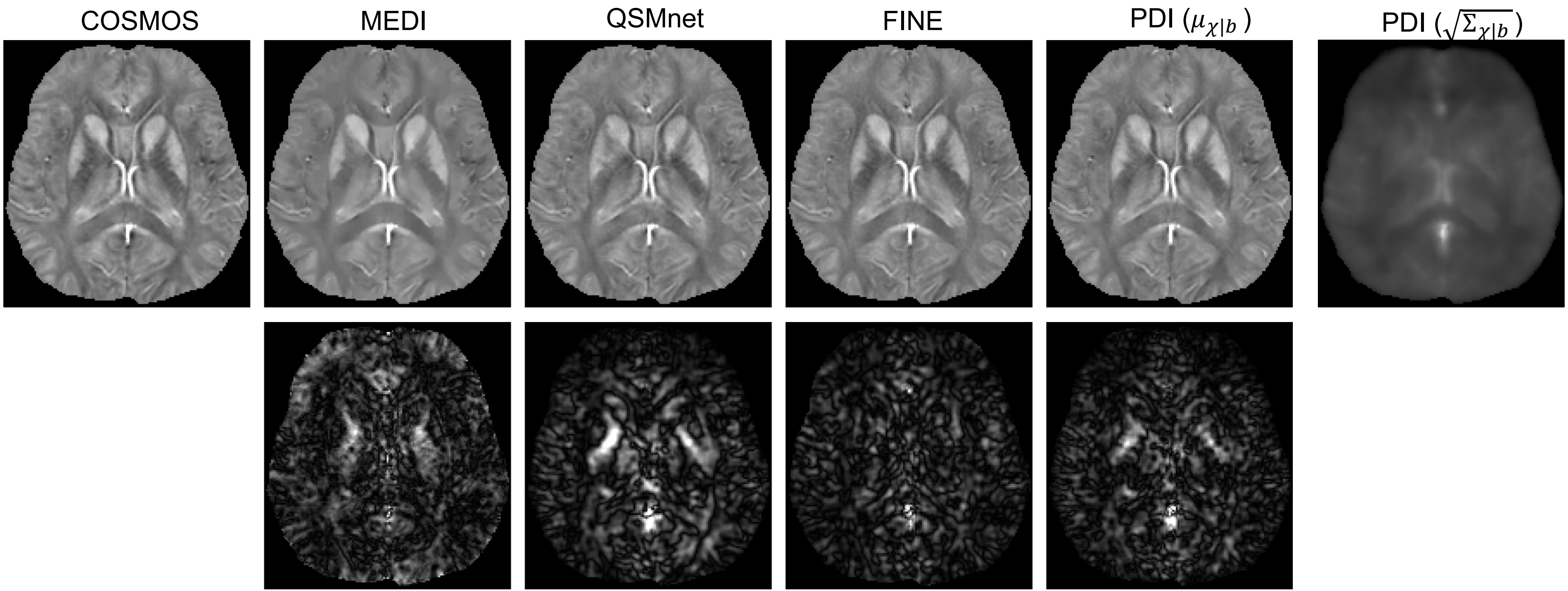
Probabilistic Dipole Inversion (PDI) network



Experimental setups

- Pre-trained on COSMOS (3D patches) → PDI
  - 4 training, 1 validation, 2 test, each having 5 orientations
- Domain adaptations on MEDI (whole brains) → PDI-VI
  - Multiple sclerosis dataset (6 training, 1 validation, 7 test)
  - Hemorrhage dataset (4 training, 1 validation, 2 test)

Healthy subject with COSMOS

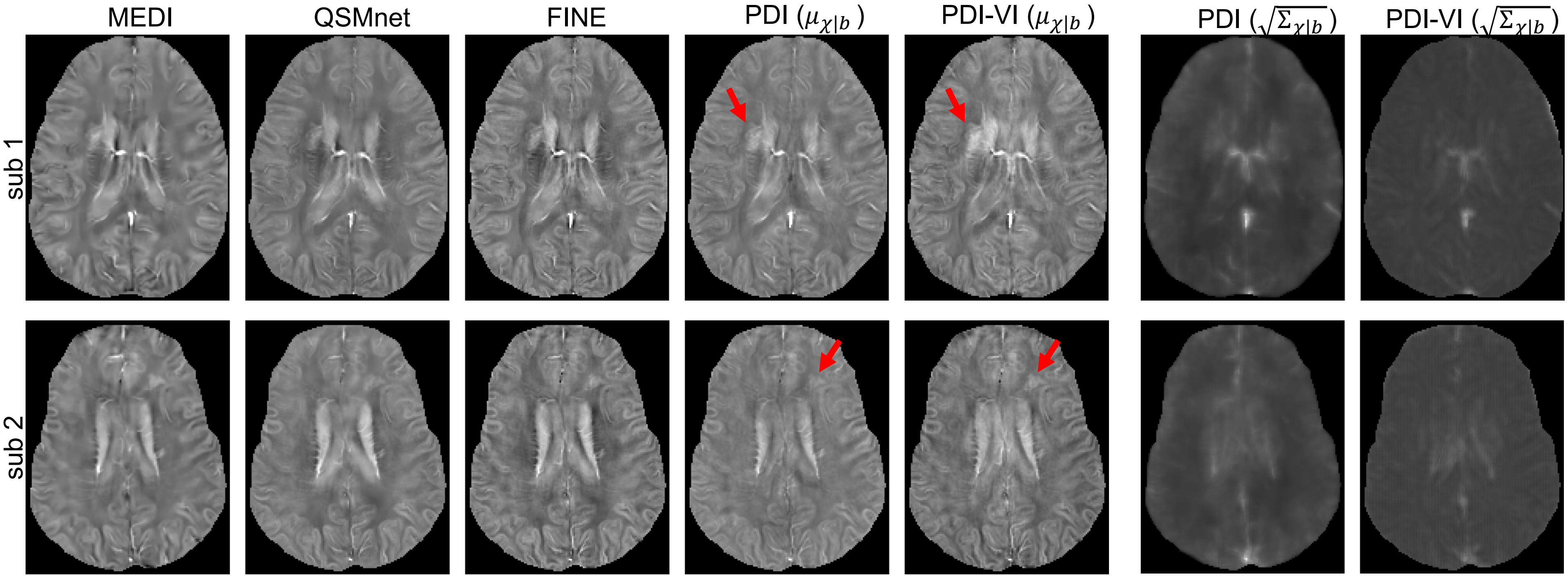


QSMnet: Yoon, Jaeyeon, et al. *Neuroimage* 179 (2018): 199-206. FINE: Zhang, Jinwei, et al. *NeuroImage* 211 (2020): 116579.

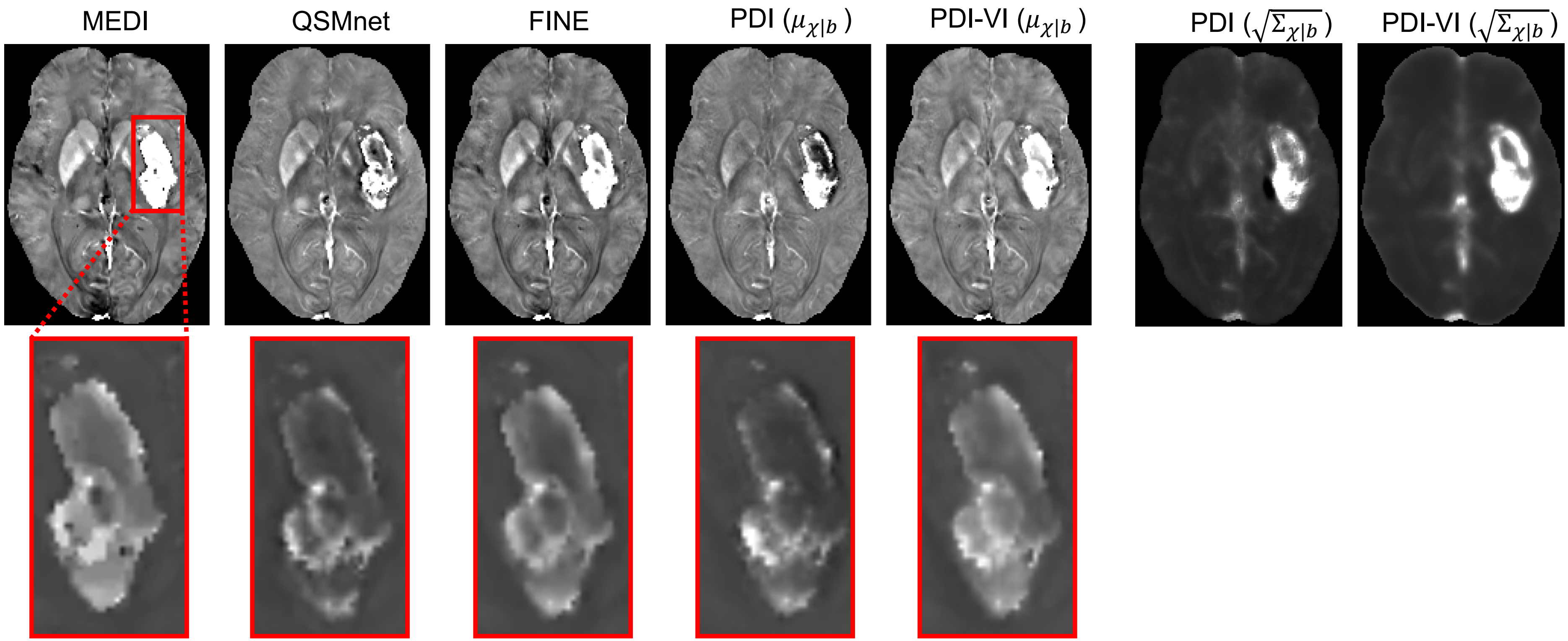
Healthy subject with COSMOS

	pSNR	RMSE	SSIM	HFEN	GPU time (s)
MEDI(Liu et al., 2012)	46.39	41.16	0.9569	31.30	17.54
FINE(Zhang et al., 2020)	48.12	33.66	0.9789	31.97	65.42
QSMnet(Yoon et al., 2018)	46.35	41.29	0.9705	43.31	0.60
PDI	47.77	35.08	0.9772	35.17	0.61

Multiple sclerosis patients

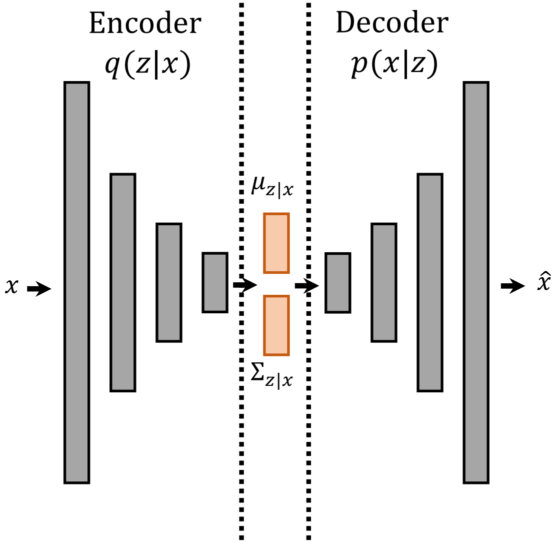


Hemorrhagic patient



Discussion: relationship to VAE

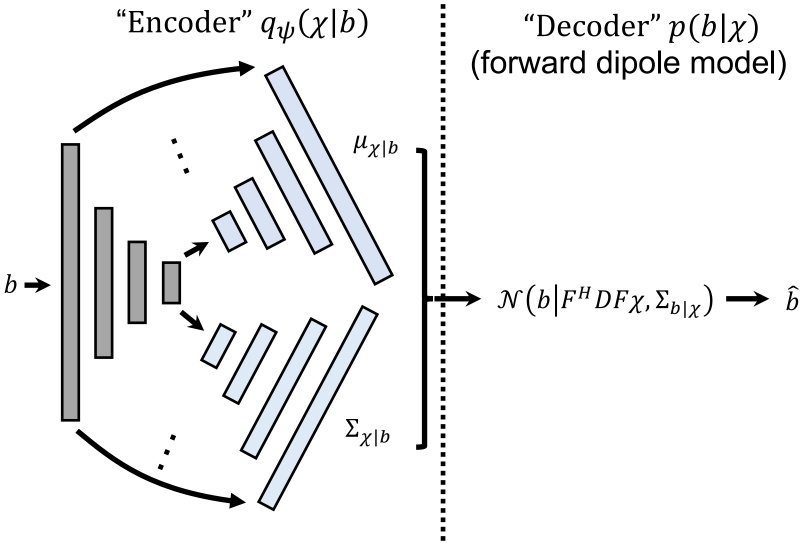
VAE architecture



loss function

$$-\underbrace{\left(\mathbb{E}_{q(z|x)}[\log p(x|z)] - KL[q(z|x) \parallel p(z)]\right)}_{\text{ELBO}}$$

PDI architecture



loss function

$$-\left(\mathbb{E}_{q_\psi}[\log p(b|\chi)] - KL[q_\psi(\chi|b) \parallel p(\chi)]\right)$$

Kingma, Diederik P., and Max Welling. "Auto-encoding variational bayes." *arXiv preprint arXiv:1312.6114* (2013).

Conclusion

- Learn a neural network parametrized distribution which yields the posterior distribution of susceptibility given input local field.
- train those parameters by fitting to the empirical distribution defined from COSMOS dataset.
- Adapt the pre-trained parameters to different domains using (amortized) variational inference.

Future work

- More expressive model family for $q_{\psi}(\chi|b)$:

invertible neural network

- Learn a prior density $p(\chi)$ instead of pre-defining:

autoregressive or VAE density estimations

Thank you

Questions